(i) Printed Pages: 3 Roll No. .....

(ii) Questions : 8 Sub. Code : 0 4 4 3

Exam. Code : 0 0 0 5

B.A./B.Sc. (General) 5th Semester

(1129)

MATHEMATICS

Paper—I : Analysis—I

Time Allowed : Three Hours] **[Maximum Marks : 30**

Note :— Attempt any **5** questions selecting at least **two** questions from each Section. All the questions carry equal marks.

SECTION—A

1. (a) Prove that the Set [0, 1] is uncountable.

   (b) If f and g are R-integrable on [0, 1] Prove that

$$\left[\int_0^1 f(x)g(x)dx\right]^2 \le \int_0^1 f^2(x)dx \cdot \int_0^1 g^2(x)dx. \qquad 3+3$$

2. (a) State and Prove Darboux's theorem.

   (b) Let F(x) be defined as

$$F(x) = \int_{-1}^{x} f(t)dt, \quad f(t) = \begin{cases} 1 & \text{for } t \le 0 \\ 1+t & \text{for } 0 < t \le 2 \end{cases}$$

Show that $\frac{d}{dx}(F(x)) = f(x) \forall x \in [-2,2]$. 3+3

3. (a) If f(x) is bounded and integrable function of x defined on [a, b] where $x = \phi(t)$ is strictly monotonic function of $t \in [\alpha, \beta]$ and possesses a bounded and integrable derivative $\phi'(t)$ on $[\alpha, \beta]$ then prove that

$$\int_a^b f(x)dx = \int_\alpha^\beta f(\phi(t))\phi'(t)dt.$$

(b) Let $f(x) = \begin{cases} 1-x & \text{if } x \text{ is irrational} \\ \sqrt{1-x^2} & \text{if } x \text{ is rational} \end{cases}$. Show that f is not Riemann integrable on [0, 1]. 3+3

4. (a) Prove that $B(m,n) = \frac{\sqrt{(m)}\sqrt{(n)}}{\sqrt{(m+n)}}$ where $m > 0$, $n > 0$.

(b) Show that $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta = \frac{\pi}{\sqrt{2}}$. 3+3

SECTION—B

5. (a) Discuss the convergence of $\int_0^\infty \frac{\cos mx - \cos nx}{x^2} dx$ where $m, n \geq 0$ using Dirichlet's Test.

(b) Discuss the convergence of $\int_0^1 \frac{(x^m + x^{-m})\log(1+x)}{x} dx$. 3+3

6. (a) Discuss the convergence of $\int_0^1 \left(\log\frac{1}{x}\right)^m dx$.

(b) Evaluate $\int_0^a \frac{\log(1+ax)}{1+x^2}dx$, $a > 0$

Hence evaluate $\int_0^1 \frac{\log(1+x)}{1+x^2}dx$. 3+3

7. (a) If f(x) is a continuous function on [0, ∞] having points of infinite discontinuity at 0 and ∞ only, $\lim_{x\to 0+} f(x) = f_0$ and $\lim_{x\to\infty} f(x) = f_1$ then prove that

$$\int_0^\infty \frac{f(ax)-f(bx)}{x}dx = (f_0 - f_1)\log\frac{b}{a}.$$

(b) Show that $\int_0^{\pi/2} \sin x \log\sin x\, dx$ is convergent with value $\log\left(\frac{2}{e}\right)$. 3+3

8. (a) Show that $\int_0^\pi \frac{\log(1+\cos\alpha\sin x)}{\sin x}dx = \frac{\pi^2 - 4\alpha^2}{4}$ for all $\alpha \in (-\pi, \pi)$.

(b) Show that $\int_0^\infty \frac{\sin x}{x}dx$ is not absolutely convergent.

3+3

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